# AP Physics C Mechanics Summer Work Packet For Mr. Mullins

Welcome all to AP Physics C Mechanics as a stand-alone course.

# Three Goals for this course

(i) Complete all material for the course by Spring Break. Emphasis on Free Response Questions (FRQ's). We hope that you retain about 60% of the material during these three quarters.

(ii) Use fourth quarter to prepare for the multiple-choice (MCQ's) portion of the exam which is 35 items in 45 minutes. We will use this time to recognize different types of MCQ's and develop strategies for each type. The MCQ's will cover all units in the course so that individual weaknesses for each student can be simultaneously addressed. We hope that you recover 60% of the remaining 40% not mastered in the first three quarters. That puts each of you at 85% for a 5.

(iii) Help students to recover how to prepare for tests that operate at a higher level than memorization. WARNING! "Look-over homework problems" = failure in this course. A direct effort to anticipate test topics and problems will need to be developed.

# **Textbook references**

I have notes already constructed for the entire course using a pdf version of <u>Fundamentals of</u> <u>Physics</u>, 9<sup>th</sup> ed., by Halliday, Resnick, and Walker. If you can find a free, legal, pdf version of this textbook online during the summer, try to download it for future references.

# **Calculator Needs and Skills**

Most of my notes from the above text will use images imported from a TI-83+ calculator with a DOS that is now outdated. We will have a set of 30 TI-Inspires in the classroom. I will use a new and improved TI-84 Plus. Plan to get a graphing calculator for use with homework. TI-84's are better for calculus and physics.

# **Geometry/Trigonometry Review**

Physics operates off a few identities among many required for calculus and precalculus. You will be given a formula sheet for all tests from me. The same sheet will be supplied on your actual AP exam in May of 2024.

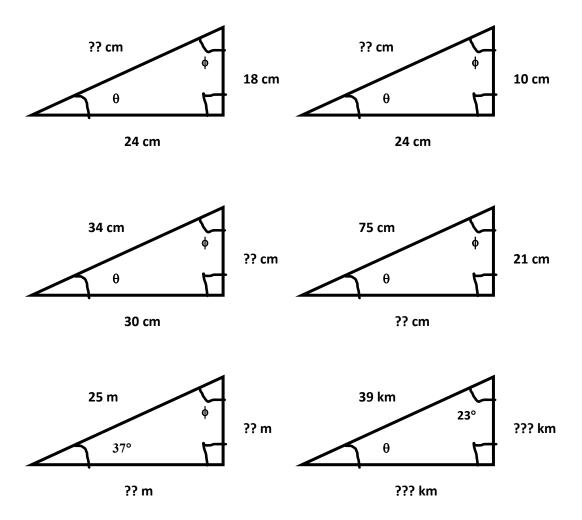
Go to <u>https://secure-media.collegeboard.org/digitalServices/pdf/ap/physics-c-tables-and-equations-list.pdf</u> and print a copy of all three pages to keep in the front of your notebook for this course. We will learn all the calculus identities during the first two weeks of the course. Be familiar with the rest of the math identities before we meet on day #1.

Identities not found on this sheet but sometimes used include (i)  $\sin^2(\theta) + \cos^2(\theta) = 1$  and (ii)  $\sin(2\theta) = 2 \cdot \sin(\theta) \cdot \cos(\theta)$  Learn to recognize the side length ratios and the interior angles for the following *Pythagorean Triple Triangles* before class begins.

(1) Triangles of side length ratios 3 by 4 by 5 are right triangles with inside angles of 37° & 53°.

- (2) Triangles of side length ratios 5 by 12 by 13 are right triangles with angles of 67° & 23°.
- (3) Triangles of side length ratios 8 by 15 by 17 are right triangles with angles of  $62^{\circ} \& 28^{\circ}$ .
- (4) Triangles of side length ratios 7 by 24 by 25 are right triangles with angles of 16° & 74°.

**<u>Pythagorean Triple Practice</u>**: For each right triangle, find the remaining sides and angles when given either two side lengths or one side length plus one angle.



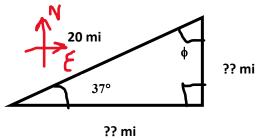
# **Vector Addition Practice**

A vector is a value that consists of a number part (magnitude) and a direction part. In most cases, the vector is given as {a number value with a unit} at \_\_\_\_\_° {some reference axis}. Example #1:

A ball is thrown at 12 m/s at an angle of 45° above the horizontal direction.

In the above example the angle is in a vertical plane so "from the vertical" or "from the horizontal" are commonly used with the horizontal reference preferred in most cases. Example #2:

Charlie walks 20 miles from home at a direction 37° North of the Eastern direction. How far is Charlie east of home? How far is Charlie west of home? Imagine Charlie walking along the hypotenuse of a right triangle with a hypotenuse of 20 miles as shown in the figure to the right.



The x part of this vector (x-component) can be found using  $x = r \cos\theta$ . Subbing shows that Charlie is at  $x = 20m \cdot \cos(37^\circ) = 16$  miles East of home.

The y part of this vector (y-component) can be found using  $y = r \sin\theta$ . Subbing shows that Charlie is at  $y = 20 \text{m} \cdot \sin(37^\circ) = 12$  miles North of home.

### Practice Problem #1

A drone starts above the flagpole at WSHS. If flies 13 meters at 22.6° North of East axis. The drone momentarily pauses to take a picture of the campus. (a) How far east of the flagpole was the picture taken? (b) How far north of the flagpole was the picture taken? The drone then flies 10 meters at 36.9° North of the western axis and lands. (c) How far east of the flagpole is the drone when it lands? (d) How far north of the flagpole is the drone when it lands? (e) How far must a person walk from the flagpole directly to the drone to retrieve the drone? *Answers:* (a) 12 m, (b) 5 m, (c) 4 m, (d) 11 m, (e)  $\sqrt{(137)}$  m

### Practice Problem #2

A sprinting spaniel leaves a tree running for 25 meters at  $16.3^{\circ}$  east of the north axis to capture a red ball. The spaniel then runs 15.3 meters at  $31.6^{\circ}$  south of the east axis to return the ball to his thrower person. How far is the person from the base of the tree? *Answer:* 25.6 m

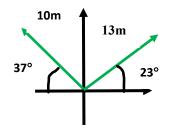
### How to prepare for a physics test problem

Write the steps that you used to get an answer to practice problem #2 using complete sentences in the correct order. *If you cannot do this part, then you are not ready for a test on this part.* 

There are two methods outlined in the following page. The components method is preferred for physics since you get a good physical sense of what is happening and since the complexity of the method does not geometrically multiply when going from adding to vectors to adding more than two vectors.

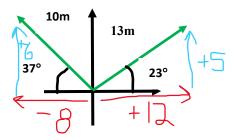
#### Method of Components Steps (Using Problem #1)

1. Draw each vector as a ray pointing away from the origin including labels where the length of the ray is the magnitude of the vector and a reference angle to one of the primary axes.

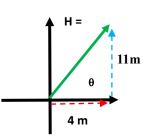


2. Make each ray the hypotenuse of a right triangle. Use opposite = hypotenuse  $\cdot \sin\theta$  and adjacent = hypotenuse  $\cdot \cos\theta$  to get the sides of each triangle. The sign of each component is determined using right direction and up direction as positive. To the left and down are (-).

For 13 m vector: x part =  $13\cos(22.6) = 12m$ . y part =  $13\sin(22.6) = 5m$ For 10 m vector: x part =  $-10\cos(36.9) = -8$ . y part =  $10\sin(36.9) = +6$  m



- 3. Combine all x parts to get a total x: For the flying drone, 12 m 8 m = +4 m (to the East) Combine all y parts to get a total y: For the flying drone, 5 m + 6 m = +11 m (to the North)
- 4. Draw the totals in a head-to-tail fashion to create the perpendicular sides of a right triangle.
- 5. The final answer is the hypotenuse that closes the right triangle. Use Pythagorean Theorem to get the magnitude of the final answer. Use SOH-CAH-TOA to get the direction of the final vector with reference from the angle closest to the origin.



H =  $\sqrt{(4^2 + 11^2)}$  = 11.7 m from the flagpole; tan  $\theta$  = 11/4 and  $\theta$  = 70° North of the east axis.

<u>Head-to-Tail Method</u> (Steps without images to save space)

- 1. Draw two vectors in a head-to-tail configuration.
- 2. Close the triangle with a 3<sup>rd</sup> ray that points from the tail of the first vector at the origin to the head of the second vector away from the origin. The 3<sup>rd</sup> ray is your final answer.
- 3. Use Geometry reasoning to determine the interior angle  $\theta$  between the two original vectors.
- 4. Use Law of Cosines to get the magnitude of the  $3^{rd}$  ray.  $\Rightarrow c^2 = a^2 + b^2 2ab \cos\theta$ .
- 5. Use Law of Sines to get the remaining angles inside of the triangle.
- 6. Use geometry reasoning to get the reference angle of 3<sup>rd</sup> vector from a primary axis.

# **Graphing Calculator Exercises**

# Analyzing a position graph

An object moving along the x-axis can have its motion described by a position vs time equation such as  $x = 6 - 5t + t^2$ . The equation calls for the object to begin 6 meters to the right of the origin with an initial speed moving left at 5 m/s. We will learn to analyze this motion through functional approaches as well as graphical approaches. Things like slope, intercepts, minimum, and maximum values can tell us some useful physics information. Your graphing calculator can quickly arrive at useful information. Get a graphing calculator. Go through the following steps.

Go to [y =] for a TI 84 graphing calculator or  $f_1$  for the TI Inspire and insert the equation for  $Y_1 = 6 - 5x + x^2$ . Your window will be  $x_{min} = 0$  and  $x_{max} = 4$  with  $x_{scl} = 1$ . The  $y_{min}$ will be -0.5 and the  $y_{max} = 6.5$ . Use of the [graph] button should show a parabola like the one in the figure to the right.

Your graphing calculator can tell you things about this graph such as the slope of a tangent line to the graph at any point.

(i) What is the slope of the graph at x = 2?
(ii) What is the slope of the graph at x = 3?
(iii) What are the x and y values at the minimum?
Answers: (i) -1, (ii) +1, (iii) ⟨2.5, -0.25⟩

You can use the button sequence of  $[2^{nd}]$ , [Trace], [6] to get slope. For the TI-Inspire, use a sub menu of "analyze graph". In both calculators, slope is not written as  $\Delta y/\Delta x$  but rather as dy/dx. The differences are not important currently. We will clarify this matter when school begins.

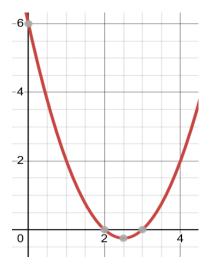
Consider an object moving along the x axis as a function of time where  $x = 4 + 3t - t^2/2$  for  $0 \le t \le 8$  s with x in meters and t in seconds. A graph is shown in the figure to the right using DESMOS. The slope of the position graph at any point represents the velocity of the object at that instant in time.

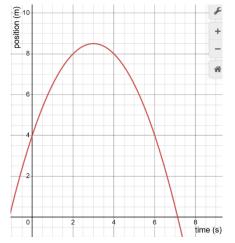
You can fool the calculator by graphing in functional mode where  $y = 4 + 3x - x^2/2$ . Just remember that the calculator's x is really our time and the calculator's y is really our position, x. Find the velocity of the object at t = 0 s, 2 s, 3 s, 4 s, and 6 s. You can find the time at which the object passes through the origin using [2<sup>nd</sup>] [Trace] {zero}.

Answers: 3 m/s, 1 m/s, 0 m/s, -1 m/s, -3 m/s.

Object passes through the origin moving to the left at 7.123 seconds.

When given x = f(t), SWBAT construct a graph in functional mode. Also, SWBAT to find the slope of the function (graph) which represents the velocity at that instant in time.





#### Analyzing a velocity vs time graph

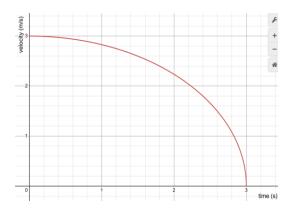
Velocity vs time graphs have several key factors that make them high probability targets on AP Physics exams. The following facts are currently stated without proof. Calculus can be used to verify these facts. However, we wish to let the graphing calculator do the work for us to begin.

- The slope at a point on a velocity vs time graph represents the acceleration at that instant in time. Acceleration in one-dimensional motion is the rate of change of speed with time. You can say "4 meters per second square" and write 4 m/s<sup>2</sup>. But think instead about 4 m/s each second or four meters per second every second.
- The area under the curve between two different time values represents the displacement of an object. The amount of displacement depends on the different in the two times. We will designate the initial time as t<sub>o</sub> and the final time as t<sub>f</sub>. The calculator will ask for these two times using the phrases- "lower limit?" and "upper limit?". The lower limit will be our initial time and the upper limit will be our final time.

# Practice Problem #1

A person on a bicycle is moving at 3 m/s when the hand brakes are applied. The velocity of the rider decreases to rest according to the equation  $v = \sqrt{9 - t^2}$  for  $0 \le t \le 3s$ . The graph is shown in the image to the right.

(a) What is the acceleration rate of the bicycle at t = 2s? (b) How far does the bike travel during the first two seconds of braking? (c) How far does the bike travel while slowing from 3 m/s to rest?



Place your equation in your graphing calculator in functional mode.  $f_1$  or  $Y_1 = \sqrt{9 - x^2}$ . Set your window according to the scales in the image above. Minimum values are at -0.5 and maximum values are about 3.4.

(a) Use  $[2^{nd}]$ , [Trace], [6], [2] to get the acceleration = -0.8944 m/s<sup>2</sup>. Interesting side note is that the units come from the concept of *slope* = *rise/run*. To get the units of slope, merely divide the y-axis units (m/s) by the x-axis units (seconds) to get slope units. Plug in units instead of values!

(b) Use  $[2^{nd}]$ , [Trace], [7] to get to the area under the graph from 0s to 2s. You will be prompted to supply input values of 0 for lower limit and 2 for upper limit. The bike travels 5.520 m in the first two seconds. Since *Area* = *height width*, you can plug in the y-units times the x-units to see that area for this graph is  $(m/s) \cdot (s) =$  meters.

(c) Use [2<sup>nd</sup>], [Trace], [7] to get to the area under the graph from 0s to 3s. You will be prompted to supply input values of 0 for lower limit and 3 for upper limit. The bike travels 7.069 m while braking to rest.

If you enjoy watching the area fill in for the TI-84 calculator, you can have a do-over. [2<sup>nd</sup>], [prgm], [1], [ENTER] will clear the shaded region and redraw the graph.

#### Practice Problem #2

A car is moving to the right at 16 m/s. As the car reaches x = 0 at t = 0, the brakes are applied bringing the car to rest in 4 seconds. The velocity vs time function is  $v = 16 - t^2$ . (a) Determine the acceleration at t = 2.0 s. (b) Determine the distance travelled in the first two seconds. (c) Determine the speed at t = 2.0 s. (d) Find the total distance traveled while braking to rest.

# Answers: (a) – 4 m/s<sup>2</sup>, (b) 29.333 m, (c) 12 m/s, (d) 42.667 m

I hope that you have a great summer. Feel free to contact me if you have any questions concerning this material. I will provide you with my cell number when you check in with me in person. Parents are always encouraged to listen to our conversations on speaker phone mode.

### Accomplishments this summer

✓ Get a graphing calculator. Learn how to use it to construct & analyze graphs using buttons [Y=], [Window], [TRACE], and [Graph]. And using the menu options for [2<sup>nd</sup>], [Trace].

\*As you can see, I will be more beneficial to you with a TI 84 rather than a TI Inspire. I am old enough that older TI-84's are just as good as a new one if you are on a budget.

- ✓ Find the slope of a graph at a designated point on the graph using  $[2^{nd}]$ , [Trace], [6].
- ✓ Find the area under the curve of a graph for a specific domain using  $[2^{nd}]$ , [Trace], [7].
- ✓ Break a vector into components using SOH-CAH-TOA
- $\checkmark$  Add vectors using the components method.
- ✓ Become familiar with common Pythagorean Triples. We use these a lot in the MCQ sections to keep the math clean.
- ✓ Change your attitude concerning test preparation from "look over" to "write down" as demonstrated in the vector addition section.
- ✓ Try to locate and download a legally obtained, free pdf version of <u>Fundamentals of Physics</u>, 9<sup>th</sup> edition by Halliday, Resnick, and Walker. The extended version will be most useful if you plan on taking some follow-up AP Physics courses.
- ✓ Print a personal copy of the AP Physics C Mechanics formula sheets. Print all 3 pages if possible, but the last two are much more important right now.